Find the least integer *n* for which p*n*(2) approximates *f*(2) with three decimal place accuracy

From

f(a+h) approximately *f*(a)+h f(a)

When h is small enough in terms of value of *f*(a) and *f*(a)

it is possible to approximate the value of

f (a+h)

For this case

let approximate the value

Of √2.1

Therefore 2.1 can be expressed as

2.1 = 2+ h where

h = 0.1

Assuming f(x) =√x

Then $f^1(x)$ =1/2√x

Therefore ,by linear approximation formula

√x+h= √x +h/2√x

And then

√2.1=√2+0.1/2√2=

√ 2.1=√2+0.035

=1.4495

Use Tylor polynomials to estimate the following to within 0.01

$e^{0.8}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$

$e^{0.8} = 1 + 0.8 + \frac{0.8^2}{2!} + \frac{0.8^3}{3!} + \ldots + \frac{0.8^n}{n!}$

$=1 + 0.8 + \frac{0.8^2}{2!} + \frac{0.8^3}{3!}$

$= 1 + 0.8 + \frac{0.64}{2} + \frac{0.212}{6}$

$= 1 + 0.8 + 0.32 + 0.0353$

$= 2.1553$

Expand as indicated

Ln (x^2)

Let x^2 be $(x-1)^2$

Where 2 is constant

= then

$\text{Ln } (x-1)^2=2\left\{(x-1)^2/(x+1)^2\right\} +1/3\left\{(x-1)^3/(x+2)^3\right\} + 1/5\left\{(x-5)^5/(x+5)^5\right\}$

For $x > 0$

For

$(a+b)^n = a^n + n/1!\, a^{n-1}b + n(n-1)/2!{*}\, a^{n-2}\, b^2$…………………………

For this case, let **1** be **a** and **2x** be **b**

Therefore,

$(1-2x)^{-3}$

$=1^{-3}+3/11{*}1^{-3}\, 2{*} + -3(-4)/2!\, {*}\, 1^{-5} +4x^2 +$ ……………..

=-1+ $\left\{(-3/1!*1^{-4}*(\_2x)\right\}$ +

$(-3(-4)/2*1^{-5}*4^{x2})$ +................................

= $-1+6x+24x^2+2$

=$24x^2+6x-1+2$

Find interval of convergence

$\sum (-1)^k (2/3)^k (x+1)^k$

$$\text{Lim} \left| \frac{(-1)^{k+1} (2/3)^{k+1} (x+1)^{k+1}}{(-1)^k (2/3)^k (x+1)^k} \right|$$

$$\text{Lim} \left| \frac{(-1)(2/3)(x+1)}{1} \right|$$

$$= \left| -2/3 (x+1) \right|$$

=-(x+1) lim 2/3

= -x-1 lim 2/3

$= -2/3 \left| x+1 \right|$ ¢ Therefore interval

$-2/3 \left| x+1 \right| < 1$ Convergence.

$$\sum \left| \frac{2^{1/k} \pi^k}{K (k+1) (k+2)} \right| (x-2)^k$$

$$\text{Lim} \left| \frac{2^{1/r} \pi^k}{K (k+1) (k+2) (k+1)} \quad (x-2)^k \right|^{1/k}$$

$$\text{Lim}_{\rightarrow 0} \left| \frac{2^{1/k} \pi^k (x-2)^{k+1}}{k (k+1)^2 (k+2)} \right|$$

$$= \left| x-2 \right| \rightarrow \frac{2^{1/k} \pi^k (x-2)^{k+1}}{k (k+1)^2 (k+2)}$$

$= 0$

Therefore, $f = 0 < 1$

Evaluation of the given limits

$$\text{Lim}\ \frac{e^x - 1 - x}{x \tan^{-1} c}$$

Using hospital rule,

$$\text{Lim}\ \frac{e^x - 1 - x}{x \tan^{-1} c} = \text{Lim}\ \frac{e^x - 1}{\text{Tan}^{-1} c}$$

As the $e^x - 1$ and $\tan^{-1}c$ tends to zero, then

$$\text{Lim}\ \frac{e^x}{\tan^{-1} c}$$

$$= \frac{1}{o} \quad = +\infty$$

Estimate within 0.01

$$\int_0^1 e^{-3x}\,dx$$

$$= \left[e^{-3x}\right]_0^1$$

$$= \left[e^{-1} - e^0\right]$$

$$= \left[0.368 - 1\right]$$

$$= -0.632$$

Reference

Karner. G and Kuich. W, (1997). “Characterizations of Abstract Families of Algebraic Power Series”.